A diagramatic solution of the NK model with fixed prices (based on Williamson's book)

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1 Introduction

The details of the mechanisms were discussed in the lectures so this file just formalises the diagrams and provides a brief sketch of the rationale behind the results. In all cases, we start from the steady state where the equilibrium is the same as that under flexible prices. Mathematically, we have the production function and labour supply and money demand. The specific form of the production function (the equation) is assumed so this is one of the key inputs; from this we obtain the demand for labour as it is just the derivative of the production function. Labour supply is obtained from the first order conditions for the household. We are not going to derive it here but just be aware that it is not assumed. Similarly, money demand originates from household optimisation where, as in the lectures, agents can purchase goods with either money or credit. All other curves are derived from the above.

The assumption in the New Keynesian (NK) model is that prices are fixed in the short-run so any effects below that differ from their RBC counterparts are temporary. In the diagrams below the subscript '1' represents the original equilibrium, before any shocks, where the values are the same as those that would arise under flexible prices.

Throughout the analysis, start with the output demand-interest rate equilibrium (output supply must adjust so it is 'explained' by the other two curves). Then just trace the dots across to the other diagrams.

1.1 A decrease in interest rates

The central bank controls the nominal interest rate. If the central bank decreases the nominal interest rate

- This leads to a one-for-one decrease in the real interest rate; we move along the Y^d curve.
- The new equilibrium is one with Y_2 , r_2 and the economy is at point b. Although not plotted, output supply must then shift and go through b.
- Once we know the value of output, we can obtain the new value of employment by making use of the production function.
- Next, we turn to the labour market. Households are always on the labour supply, which shifts up (contracts) due to the change in the interest rate. At the same time, we know how much labour firms need in order to produce Y_2 so labour demand will have to shift and go through b (again, this is not plotted). Therefore, real wages rise (in order to induce households to supply the extra labour that is required to produce the additional output).
- The decrease in interest rates and increase in income lead to an increase in money demand so *M* rises (supply is endogenous).

1.2 Expansionary fiscal policy

Now we analyse the effects of an increase in G and apply the same logic as above.

- We begin the analysis considering output demand and the interest rate.
- Y^d rises (even if consumption falls) but interest rates do not change by assumption. Therefore, the increase in G causes output to rise.
- Knowing that output equals Y_2 , we trace the dots back to the production function to obtain the value of N_2 . Given that technology has not changed, more output can only be produced with more labour so we know that N_2 has risen.
- Next, we turn to the labour market noting that labour supply increases (due to the rise in lifetime taxes) but we also know that $N = N_2$. Therefore, labour demand has to go through b:

intersecting the new labour supply schedule at the new level of employment. Consequently, real wages rise.

• As interest rates have not changed but income has risen, money demand increases.

1.3 Persistent increase in TFP

This application is more subtle. We assume that both z and z' increase.

- The first thing to note is that output demand, Y^d shifts up due to the increase in z'. The reason is that the greater marginal product of capital in the next periods induces a rise in investment. At the same time, the interest rate has not changed by assumption so overall, output rises to Y_2 and interest rates do not change.
- We work backwards to find what value of employment is consistent with producing Y_2 . The new production function has shifted due to the increase in z; we therefore observe that to produce the new level of output less labour is required, which is now equal to N_2 .
- Turning to the labour market, the labour supply schedule has not shifted but we know that $N = N_2$. Therefore (not plotted) labour demand contracts and this causes real wages to fall.
- Lastly, money demand rises given the increase in income.



Figure 1: Effects of change in nominal interest rate



Figure 2: Effects of increase in ${\cal G}$



Figure 3: Effects of a persistent increase in z