Solving representative agent models

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The one-period model

Recall the one-period model. We have three types of agents:

1. households
2. firms
3. government

Given our assumption of flexible prices and perfect competition

- Households take wages as given
- As do firms

We can solve via the competitive equilibrium (CE) or using the social planner (SP)

Households

Households aim to maximise

$$U = U(C, L)$$

subject to

$$C = wN + \pi - T \quad (1)$$

$$N + L = 1$$

Setting up the Lagrangean we have

$$\mathcal{L} = U(C, L) + \lambda \left[w(1 - L) + \pi - T - C \right]$$

focs are

$$U_C = \lambda$$

$$U_L = w\lambda$$

Thus

$$U_L = U_C w \tag{2}$$

Firms

They aim to maximise profits,

$$\pi = zF(\bar{K}, N) - wN \tag{3}$$

foc is

$$MPN = w \tag{4}$$

Government

As this is a one-period model,

$$G = T \tag{5}$$

If we combine the budget constraints (household and government) with the profit function, we have

$$Y = C + G$$

This is the aggregate resource constraint ($Y = ZF(\bar{K}, N)$)

Recall that in a standard set-up, an increase in Z

- Raises the MPN, increasing labour demand and pushing up the real wage
- If $SE > IE$, the increasing real wage raises labour supply
- Overall, N goes up (so L must fall)
- As the production function has Z and N rising, output (GDP) rises
- As G has not risen, the increase in Y corresponds with a one-for-one increase in C

Solving using the SP

This is easier. The SP aims to maximise

$$U(C, L)$$

s.t.

$$zF(\bar{K}, N) = C + G$$

so the only choices are N and C.

FOCS

$$U_C = \lambda$$

$$U_L = MPN\lambda$$

Yielding $U_L = U_C MPN$

Combine this with the aggregate resource constraint and we have two equations with two unknowns

When working with a quantitative example we look for the same insights **but** you have to consider the explicit answers that the model yields

These may differ from what you expect

An example

$$U = \ln C + bL$$

$$Y = zK^{\alpha}N^{1-\alpha}$$

$$G = 0$$

Important considerations when solving and interpreting a model

- What information are you given, what are we solving for?
- What is endogenous/exogenous?

From the equations above, we have

$$U_C = \frac{1}{C}$$

$$U_L = b$$

So

$$\frac{U_L}{U_C} = bC = w$$

and

$$MPN \equiv (1-\alpha)\frac{Y}{N} = w$$

Combining, we have

$$bC = (1-\alpha)\frac{Y}{N}$$

Use

$$Y = C + G = C$$

Then

$$bY = (1-\alpha)\frac{Y}{N}$$

$$N = \frac{(1-\alpha)}{b}$$

Having found the value of N, we have

$$C = Y = zK^{\alpha} \left(\frac{1 - \alpha}{b} \right)^{1-\alpha}$$

In this model, an increase in Z

- Raises output and consumption
- But what about N (and thus L)?

Solving using the SP

We had

$$U_L = U_C MPN$$

$$bC = (1-\alpha)\frac{Y}{N}$$

$$N = \frac{1-\alpha}{b}$$

Exactly the same as under the CE