A diagramatic solution of the NK model with inflation (loosely based on Williamson's book)

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1 Introduction

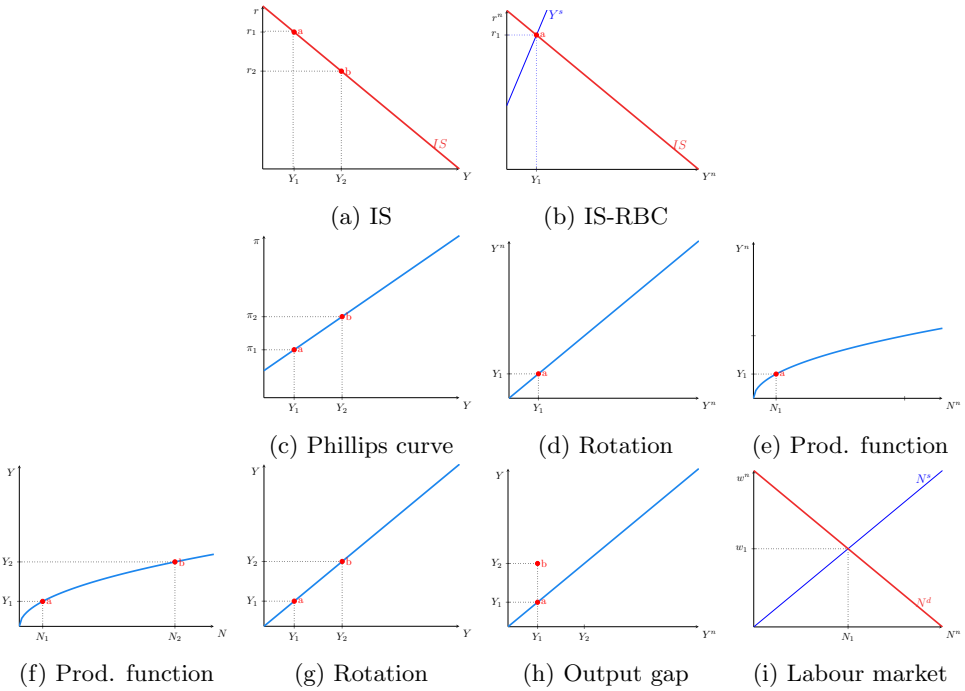
Here we extend the analysis in Williamson's book by plotting the several figures simultaneously. The left half of below represents actual outcomes in the presence of sticky prices, while the right half shows the flexible-price equivalent values (these have the superscript n to denote natural values). We assume that expected inflation is equal to the inflation target in the analysis that follows.[1]

Before any shocks, the economy is at point a so that flexible and sticky price outcomes are the same (the output gap is zero). A sudden decrease in the nominal interest rate by the central bank

- lowers the real interest rate and as panel 2a shows, output increases (due to a rise in consumption and investment);
- this increase in output leads to an increase in inflation (panel 1c);
- nothing has happened to the production function so the increase in output is obtained by increases in employment;
- the monetary policy action affected the real interest rate only because of the sticky-price assumption but the flexible-price version of the model (the natural values) remain unaffected. Hence, the right half of the figure is unchanged. As a result, Y^n remains at a but Y has risen: output is greater than its flexible price counterpart. In other words, monetary policy is non-neutral.

In summary, an unexpected temporary decrease in the nominal interest rate leads to an increase in output and inflation but leaves the flexible-price level of output unaffected. You can easily work out the effects on wages, employment, etc.

[1] I did not include the labour market under sticky prices but it is easy to see what is happening there.



(a) IS (b) IS-RBC

(c) Phillips curve (d) Rotation (e) Prod. function

(f) Prod. function (g) Rotation (h) Output gap (i) Labour market

2 A shock to the natural rate of interest

Consider a shock, such as an increase in current TFP, that shifts output supply downwards so that both the natural rate of interest falls and Y^n rises. As we are assuming that this shock is temporary, future TFP is expected to remain at its original level so Y^d, output demand, does not shift. The changes in the natural values/rates are presented by the move from a to b in the figure. Let us start with the flexible-price side of the model (the right half of the figure) as it is independent of the sticky price version.

- The flex-price side of the model shows that the increase in output and fall in the interest rate is accompanied by an increase in employment (both supply and demand increase, driven by the latter). The higher level of technology leads to a steeper production function and thus its derivative rises (the demand for labour shifts up), resulting in an increase in the real wage.
- At the same time, equilibrium in the goods market implies that the natural rate of interest falls to r_2^n and this, in turn, results in a decrease labour supply.
- Overall, the flexible-price values for output, employment, wages and the interest rate are given by Y_2, N_2, w_2 and r_2, respectively, which correspond to points 'b' in the diagrams.[2]

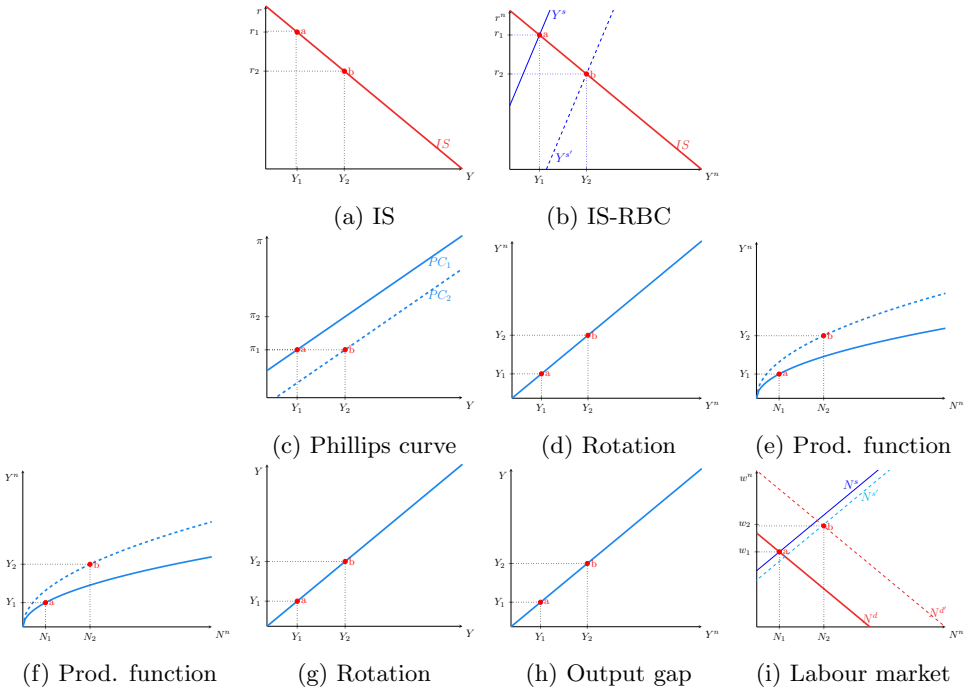
Now we can turn to the sticky-price side of the model.

- The IS remains unchanged, just as under flexible prices.
- The Phillips curve (PC) shifts down. The reason is that one of its intercepts is the flexible price level of output, which has now risen. By how much do is it shift? The PC is given by

$$i = bi' + a(Y - Y_m)$$

Where i is inflation relative to target. Originally, we had $i = 0$ so that $Y = Y_m = Y_1$, corresponding to point a in the PC. After the shock, if Y were to increase to match Y_m, we would

[2] Again, this only applies to the subplots in the right half of the figure.



(a) IS (b) IS-RBC

(c) Phillips curve (d) Rotation (e) Prod. function

(f) Prod. function (g) Rotation (h) Output gap (i) Labour market

again have $i = 0$ (assuming expected inflation does not change). We already have Y_2 from the RBC side of the model so matching this with $i = i_1$, we now have a new point on the shifted PC, whose slope has not changed.

- This completes the derivation of the NK model. The bottom left subplot includes the labour market again just so that we can determine the effects on actual employment once we know what the policy maker does to the interest rate.

In understanding the effects of the TFP shock, we already know what will happen to the natural values of all the endogenous variables, as the equilibrium is at point a, but this applies to the right half of the figure only. What about actual values? Here, the response of the central bank to the shock is key. We shall consider two possibilities (you could assume some other reaction).

Assume that the central bank ensures that the real interest rate tracks its natural rate one-for-one.

- In this case, the actual interest rate, r, falls to r_2, hence being equal to r^n. In the IS, whcih has not shifted, this implies an increase in actual output to Y_2 but this is exactly the new natural level of output, which implies an output gap of zero.

- Noting that both the flexible price level of output has risen (shifting the PC to PC_1) and that $Y = Y_2$, the new equilibrium in (Y, π) space is at b, where inflation is still equal to its target and output has risen to match its flexible-price level, so that the the output gap is still zero.

- The discussion above shows that in response to a shock that affects the IS, there is no trade-off between stabilising the output gap and stabilising inflation.

Now assume instead that the central bank keeps the interest rate unchanged at $r = r_1$. In this case, $r > r^n$. While the level of output does not change, given that its flexible price level has, we have a recession. Using this in the Phillips curve (PC_2) we note that it implies a fall in the inflation rate (just move down vertically from a in Panel (a)). Taking this to the labour market (bottom left), we would have that employment falls below N_1.