Monetary neutrality in OLG models

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1 Introduction

It is often stated that the standard OLG model with money exhibits monetary neutrality. Here we consider two versions of the benchmark model to show that who gets the monetary injection is crucial. The models are essentially identical bar for one minor detail.

Agents live for two period, they receive an endowment y when young and can sell their endowment for money to the old at a price ν_t. Population is denoted by $N_t = N$ (assumed constant) and the money supply is M_t.

Throughout, the utility function of an agent born in period t is

$$U_t = \ln c_{1t} + \ln c_{2,t+1} \tag{1}$$

2 The old get the transfer

The generation t agent faces the budget constraints

$$\begin{aligned} c_{1t} + \nu_t m_t &= y \\ c_{2,t+1} &= \nu_{t+1} m_t + a_{t+1} \end{aligned} \tag{2}$$

Where a_{t+1} is the transfer that agents receive from the governmet when old.

We can write the equations above as a lifetime budget constraint

$$c_{1t} + \frac{\nu_t}{\nu_{t+1}} \left(c_{2,t+1} - a_{t+1}\right) = y \tag{3}$$

Setting up the Lagrangean we can obtain the first order conditions for consumption in each period

$$\begin{aligned} \frac{1}{c_{1t}} &= \lambda \\ \frac{1}{c_{2,t+1}} &= \frac{nu_t}{\nu_{t+1}} \lambda \end{aligned} \tag{4}$$

Combining,

$$c_{1t} = \frac{\nu_t}{\nu_{t+1}} c_{2,t+1}$$

Substitute into the budget constraint and we obtain the desired levels of consumption in each period

$$\begin{aligned} c_{1t} &= \frac{1}{2} \left(y + \frac{\nu_t}{\nu_{t+1}} a_{t+1} \right] \\ c_{2,t+1} &= \frac{1}{2} \left[\frac{\nu_{t+1}}{\nu_t} y + a_{t+1} \right] \end{aligned} \tag{5}$$

The demand for money balances can be obtained from the first period budget constraint

$$m_t \nu_t = y - c_{1t} = \frac{1}{2}\left[y - \frac{\nu_t}{\nu_{t+1}} a_{t+1}\right]$$

2.1 Money market equilibrium

Aggregate money demand is $N m_t$. Combining this with the money supply we can find the 'price of money' that will clear the market

$$M_t = N m_t = \frac{1}{2}\left[\frac{y - \frac{\nu_t}{\nu_{t+1}} a_{t+1}}{\nu_t}\right]$$

$$\nu_t = \frac{1}{2}\frac{N}{M_t}\left[y - \frac{\nu_t}{\nu_{t+1}} a_{t+1}\right] \tag{6}$$

2.2 The government's budget constraint

We assume no government spending. Any new money is distributed to households as transfers (to the current old). In other words,

$$M_t - M_{t-1} = N a_t$$

2.3 Equilibrium with a constant money supply

If M is constant, we then have that

$$\begin{aligned} a_t &= a = 0 \\ \nu_t &= \nu = \frac{1}{2}\frac{N}{M} y \\ c_{1,t} &= c_1 = \frac{1}{2} y \\ c_{2,t+1} &= c_2 = \frac{1}{2} y \end{aligned}$$

And since $P_t = \nu_t^{-1}$,

$$\begin{aligned} P_t &= \frac{2M}{N} y \\ \Pi &= 1 \end{aligned}$$

Given a constant money supply, there is a unique price level and the net inflation rate equals zero.

2.4 A one-off increase in the money supply

Next, assume that the money supply as been equal to $M_t = M_0$ up until period t and then it permanently increases to M_1.[1]

Assume that

$$M_1 = zM_0$$

From the government's budget constraint we have (use the 1 subscripts to denote the new levels, yet to be determined)

$$\begin{aligned} (M_t - M_{t-1})\nu_t &= Na_t \\ (z-1)M_0\nu_1 &= Na_1 \end{aligned} \tag{7}$$

The market clearing condition for the money market was

$$\nu_t = \frac{1}{2}\frac{N}{M_t}\left[y - \frac{\nu_t}{\nu_{t+1}}a_{t+1}\right]$$

As the increase in M is one-off, whatever happens to ν_1 will be permanent. In other words, $\nu_2/\nu_1 = 1$. Moreover, since no more new money will be created, transfers after the current period will equal zero. Thus the equation above can be re-written as

$$\begin{aligned} \nu_1 &= \frac{1}{2}\frac{N}{zM_0}\left[y - a_2\right] \\ \nu_1 &= \frac{1}{2}\frac{N}{zM_0}y \end{aligned} \tag{8}$$

And $\nu_s = \nu_1 \forall s \geq t$.

Notice the following:

- $\frac{\nu_1}{\nu_0} = \frac{1}{z}$, or alternatively, $\Pi = z$. Prices have moved one-for-one with the increase in the money supply.
- As $\nu_2/\nu_1 = 1$ and so on, the effect on inflation is purely temporary. The economy jumps to a new price level and it stays there, hence this is an example of the quantity theory.

2.5 Is money neutral?

Under monetary neutrality, real variables are unaffected by the quantity of money. To see this, we need to determine the effects on c_{1t} and c_{2t} (the current old) of the change in M.

Firstly, note that from the government's budget constraint

$$\begin{aligned} a_1 &= (z-1)\frac{M_0}{N}\nu_1 \\ &= \left(\frac{z-1}{z}\right)\frac{1}{2}y \end{aligned} \tag{9}$$

[1] This was unexpected by the current old when young.

which produces $a_1 = 0$ in the case where $z = 1$, as expected.

From the equation for desired consumption by the current young we have ($a_{t+1} = 0$ as there will be no more transfers after the current period

$$
\begin{aligned}
c_{1t} &= \frac{1}{2}\left(y + \frac{\nu_t}{\nu_{t+1}} a_{t+1}\right) \\
c_{1t} &= \frac{1}{2} y
\end{aligned}
\tag{10}
$$

which is the same result as above.

For the current old, the answer is more subtle. For them, consumption is given by (use a prime, to denote the new variables compared to the benchmark above)

$$
\begin{aligned}
c'_{2t} &= \nu_t m_{t-1} + a_t \\
c'_{2t} &= \nu_1 m_0 + a_1 \\
c'_{2t} &= \nu_1 \frac{1}{2} \frac{y}{\nu_0} + \frac{1}{2} y + \left(\frac{z-1}{z}\right) y \\
c'_{2t} &= \frac{1}{2}\left(\frac{\nu_1}{\nu_0} + \frac{z-1}{z}\right) y \\
c'_{2t} &= \frac{1}{2} y
\end{aligned}
\tag{11}
$$

where we have used (9) and $m_t = M_t/N$.

In other words, the monetary injection has reduced the value of the current old's asset, their real money balances. However, at the same time the monetary injection was used to transfer resources (the new money) to the current old again. Or put differently, the current old receive more money and this leads to an increase in the price level, so that in real terms they are no better off.

In summary, we have seen that with the old receiving the transfers, a change in the money supply has purely nominal consequences.

3 The young receive the transfer

The model and the required steps are almost identical to those in the previous section. The key change is that the budget constraints when young and old, respectively, are now

$$
\begin{aligned}
c_{1t} + \nu_t m_t &= y + a_t \\
c_{2,t+1} &= \nu_{t+1} m_t
\end{aligned}
$$

The only difference is the location (and hence the timing) of the transfer.

The first order conditions are the same. Combining these with the lifetime budget constraint we have the desired levels of consumption

$$
\begin{aligned}
c_{1t} &= \frac{1}{2}\left(y + a_t\right) \\
c_{2,t+1} &= \frac{1}{2}\frac{\nu_{t+1}}{\nu_t}\left(y + a_t\right)
\end{aligned}
\tag{12}
$$

The demand for real money balances is then

$$\nu_t m_t = \frac{1}{2}\left(y + a_t\right) \tag{13}$$

Money market equilibrium now implies

$$\nu_t = \frac{N}{M_t}\left(\frac{y + a_t}{2}\right) \tag{14}$$

and from the government's budget constraint

$$\left(M_t - M_{t-1}\right)\nu_t = N a_t$$

3.1 A constant money supply

With $z = 1$ you can check that we obtain exactly the same results as before. This should be obvious as $z = 1$ implies that $a = 0$ so that in the absence of transfers both models are identical.

3.2 A one-off increase in the money supply

As before, assume that the money supply has been constant at M_0 and it unexpectedly increases permanently to a new level M_1 in the current period, with $M_1 = zM_0$.

First, the government's budget constraint implies

$$(z - 1)M_0\nu_1 = N a_1 \tag{15}$$

The money market equilibrium condition (14) is now

$$\nu_1 = \frac{N}{zM_0}\left(\frac{y + a_1}{2}\right) \tag{16}$$

Combining these two we can solve for ν_1 and a_1:

$$\begin{aligned}
\nu_1 &= \frac{N}{zM_0}\left(\frac{y+a_1}{2}\right) \\
\nu_1 &= \frac{N}{zM_0}\frac{y}{2} + \frac{1}{2}\frac{N}{zM_0}(z-1)\frac{M_0}{N}\nu_1 \\
\nu_1 &= \frac{N}{zM_0}\frac{y}{2} + \frac{(z-1)}{2z}\nu_1 \\
(2z-z+1)\,\nu_1 &= \frac{N}{M_0}y \\
\nu_1 &= \frac{N}{M_0}\left(\frac{1}{z+1}\right)y
\end{aligned} \tag{17}$$

and

$$a_1 = \frac{z-1}{z+1}y \tag{18}$$

As a check, if $z = 1$, then $\nu_1 = \nu_0$, while $a_1 = 0$.

As the change is one-off and permanent, $\nu_2 = \nu_1$ and so on. What does this imply for the rate of return on money and for inflation (in the current period only, as after that both equal 1)?

$$\begin{aligned}
\frac{\nu_1}{\nu_0} &= \frac{2}{1+z} \\
\Pi &= \frac{1+z}{2}
\end{aligned} \tag{19}$$

3.3 Is money neutral?

Now we perform the same exercise: to determine the current consumption levels by the young and by the old.

For the current young, we have can use their desired consumption (plus the solution for a_1)

$$\begin{aligned}
c_{1t} &= \frac{1}{2}(y+a_1) \\
c_{1t} &= \frac{1}{2}y\left(1+\frac{z-1}{z+1}\right) \\
c_{1t} &= \frac{z}{1+z}y
\end{aligned} \tag{20}$$

And for the current old, using their budget constraint (but in period t) and recalling that the endowment, $a_{t-1} = 0$

$$\begin{aligned}
c'_{2,t} &= \frac{1}{2}\frac{\nu_1}{\nu_0}(y+0) \\
c'_{2,t} &= \frac{1}{1+z}y
\end{aligned} \tag{21}$$

As a check, you should find that $c_{1t} + c'_{2,t} = y$.

Before, we had that consumption by the young was half of the endowment. Now it is $z/(1+z) = 1/(1+1/z) > 1/2$. In other words, the monetary injection has lead to consumption by the current young (and this generation only) benefitting from the monetary stimulus. At the same time, $c'_{2,t} < c_{2,t}$: the current old are worse off as their monetary holdings have lost value and so they are poorer. In this case, the monetary stimulus raised the price level but the extra money was not received by the agents holding the old money. The monetary injection has been redistributive and the effects have not been neutral: consumption by the young rose while that by the old fell. In all subsequent periods, the allocation between the young and the old will be the same as before the monetary injection; the only difference will be a higher P and a lower ν.